RESEARCH ARTICLE

REVAN INDICES AND THEIR POLYNOMIALS OF CERTAIN RHOMBUS NETWORKS

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**ABSTRACT**

In this paper, we introduce the first and second Revan polynomials of a molecular graph. We compute the first and second Revan indices and their polynomials of rhombus silicate and rhombus oxide networks. Also we determine the first Revan vertex index and third Revan index and their polynomials of rhombus silicate and rhombus oxide networks.

**Key words:** Revan indices, Rhombus silicate network, Rhombus oxide network.

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**INTRODUCTION**

We consider only finite connected undirected without loops and multiple edges. Let \( G \) be a graph with vertex set \( V(G) \) and edge set \( E(G) \). Let \( d(v) \) denote the number of vertices adjacent to \( v \). Let \( \Delta(G)(\delta(G)) \) denote the maximum (minimum) degree among the vertices of \( G \). The Revan vertex degree of a vertex \( v \) in \( G \) is defined as \( r_v(v)=\Delta(G) + \delta(G) - d_v(v) \). The Revan edge connecting the Revan vertices \( u \) and \( v \) will be denoted by \( uv \). For other undefined notations and terminologies, we refer [Kulli, 2012].

A molecular graph is a graph whose vertices correspond to the atoms and the edges to the bonds. Chemical graph theory has an important effect on the development of chemical sciences. A single number that can be used to characterize some property of the graph of molecular is called a topological index. Several such topological indices have been considered in Theoretical Chemistry and have found some applications, especially in QSPR/QSAR study, see [Todeschini and Consonni, 2009].

In [Kulli, 2017], the first and second Revan indices of a graph \( G \) are defined as

\[
R_1(G) = \sum_{uv \in E(G)} r^G_v(u) + r^G_v(v), \\
R_2(G) = \sum_{uv \in E(G)} r^G_v(u)r^G_v(v).
\]

Considering Revan indices, we define the first and second Revan polynomials of a graph \( G \) as

\[
R_1(G,x) = \sum_{uv \in E(G)} x^{r^G_v(u)+r^G_v(v)}, \\
R_2(G,x) = \sum_{uv \in E(G)} x^{r^G_v(u)r^G_v(v)}.
\]

The first Revan vertex index [Kulli, 2017] of a graph \( G \) is defined as

\[
R_{v1}(G) = \sum_{u \in V(G)} r^G_v(u^2).
\]

The third Revan index [Kulli, 2017] of a graph \( G \) is defined as

\[
R_3(G) = \sum_{uv \in E(G)} \left|r^G_v(u) - r^G_v(v)\right|.
\]

Considering the first Revan vertex index, we define the first Revan vertex polynomial as

\[
R_{v1}(G,x) = \sum_{u \in V(G)} x^{r^G_v(u)}.
\]
Considering the third Revan index, we define the third Revan polynomial as

\[ R_3(G) = \sum_{uv \in E(G)} x^{d_G(u) - d_G(v)} \]

Recently other Revan indices were studied, for example, in (Kulli 2017) and also many topological indices were studied, for example, in (Kulli, 2016; Kulli, 2017). In this paper, some Revan indices and their polynomials of rhombus silicate networks and rhombus oxide networks are determined. For rhombus silicate networks and rhombus oxide networks see [19] and references cited therein.

Results for Rhombus Silicate Networks

We consider a family of rhombus silicate networks. A rhombus silicate network is symbolized by \( RHSL_n \). A 3-dimensional rhombus silicate network is depicted in Figure 1.

\[ \text{Figure 1. A 3-dimensional rhombus silicate network} \]

In the following theorem, we compute the first Revan vertex index and its polynomial of rhombus silicate network.

**Theorem 1.** Let \( G \) be the graph of rhombus silicate network \( RHSL_n \). Then

\[ R_0(RHSL_n) = 99n^2 + 126n. \]

\[ R_0(RHSL_n, x) = (2n^2 + 4n)x^6 + (3n^2 - 2n)x^3. \]

**Proof:** Let \( G \) be the graph of \( RHSL_n \). By algebraic method, we obtain that \( G \) has \( 5n^2 + 2n \) vertices. From Figure 1, it is easy to see that the vertices of \( RHSL_n \) are either of degree 3 or 6. Thus \( \Delta(G) = 6, \delta(G) = 3 \). We partition \( V(RHSL_n) \) into two sets, vertices of degree 3 and 6 respectively.

\[ V_3 = \{ u \in V(G) | d_G(u) = 3 \}, |V_3| = 2n^2 + 4n. \]
\[ V_6 = \{ u \in V(G) | d_G(u) = 6 \}, |V_6| = 3n^2 - 2n. \]

Clearly, we have \( \Delta(G) + \delta(G) = 9 \). Thus \( r_G(u) = 9 - d_G(u) \).

Thus there are two types of Revan vertices as follows.

\[ V_{r3} = \{ u \in V(G) | r_G(u) = 3 \}, |V_{r3}| = 3n^2 - 2n. \]

1) To compute \( R_{01}(RHSL_n) \), we see that

\[ R_1(G) = \sum_{uv \in E(G)} x^{r_G(u)} + \sum_{uv \in E_{r6}} x^{r_G(u)} + \sum_{uv \in E_{r3}} x^{r_G(u)} \]

\[ = \left( 2n^2 + 4n \right) 6^2 + \left( 3n^2 - 2n \right) 3^2 = 99n^2 + 126n. \]
2) To compute $R_0(RHSL_n, x)$, we see that

$$R_0 (G, x) = \sum_{uv \in E(G)} x^{e_u (uv)} = \sum_{uv \in E_{r_3}} x^{e_u (uv)} + \sum_{uv \in E_{r_6}} x^{e_u (uv)}$$

$$= (2n^2 + 4n)x^6 + (3n^2 - 2n)x^3$$

$$= (2n^2 + 4n)x^6 + (3n^2 - 2n)x^9.$$ 

We compute the value of $R_1(RHSL_n, x)$, $R_2(RHSL_n, x)$, $R_3(RHSL_n, x)$ for rhombus silicate networks.

**Theorem 2.** Let $RHSL_n$ be the rhombus silicate network. Then

$$R_1(RHSL_n) = 90n^2 + 36n. \quad (1)$$

$$R_2(RHSL_n) = 162n^2 + 144n + 18. \quad (2)$$

$$R_3(RHSL_n) = 18n^2 + 12n - 12. \quad (3)$$

**Proof:** Let $G$ be the graph of rhombus silicate network. By calculation, we obtain that $G$ has $12n^2$ edges. In $RHSL_n$, by algebraic method, there are three types of edges based on the degree of the end vertices of each edge as follows:

$$E_{r_3} = \{ uv \in E(G) \mid d(u) = d(v) = 3 \}, \quad |E_{r_3}| = 4n + 2.$$ 

$$E_{r_6} = \{ uv \in E(G) \mid d(u) = 6, d(v) = 3 \}, \quad |E_{r_6}| = 6n^2 + 4n - 4.$$ 

$$E_{r_6} = \{ uv \in E(G) \mid d(u) = 6, d(v) = 6 \}, \quad |E_{r_6}| = 6n^2 - 8n + 2.$$ 

Clearly we have $\Delta (G) = 6$, $\delta (G) = 3$. Thus $r_6(u) = 9 - d(u)$. Thus there are three types of Revan edges based on the degree of the end revan vertices of each revan edge as follows:

$$RE_{r_3} = \{ uv \in E(G) \mid r_3(u) = r_3(v) = 3 \}, \quad |RE_{r_3}| = 6n^2 - 8n + 2.$$ 

(1) We compute $R_1(RHSL_n)$, we see that

$$R_1(G) = \sum_{uv \in E(G)} [r_G(u) + r_G(v)] = \sum_{RE_{r_3}} [r_G(u) + r_G(v)] + \sum_{RE_{r_6}} [r_G(u) + r_G(v)] + \sum_{RE_8} [r_G(u) + r_G(v)]$$

$$= 12(4n + 2) + 9(6n^2 + 4n - 4) + 6(6n^2 - 8n + 2)$$

$$= 90n^2 + 36n.$$ 

(2) To compute $R_2(RHSL_n)$, we see that

$$R_2(G) = \sum_{uv \in E(G)} r_G(u) r_G(v) = \sum_{RE_{r_3}} r_G(u) r_G(v) + \sum_{RE_{r_6}} r_G(u) r_G(v) + \sum_{RE_8} r_G(u) r_G(v)$$

$$= 36(4n + 2) + 18(6n^2 + 4n - 4) + 9(6n^2 - 8n + 2)$$

$$= 162n^2 + 144n + 18.$$ 

(3) To compute $R_3(RHSL_n)$ we see that

$$R_3(G) = \sum_{uv \in E(G)} |r_G(u) - r_G(v)| = \sum_{RE_{r_3}} |r_G(u) - r_G(v)| + \sum_{RE_{r_6}} |r_G(u) - r_G(v)| + \sum_{RE_8} |r_G(u) - r_G(v)|$$

$$= 0(4n + 2) + 3(6n^2 + 4n - 4) + 0(6n^2 - 8n + 2) = 18n^2 + 12n - 12.$$ 

In the following theorem, we compute the value of $R_4(RHSL_n, x)$, $R_2(RHSL_n, x)$, $R_3(RHSL_n, x)$ for rhombus silicate networks.

**Theorem 3.** Let $RHSL_n$ be the rhombus silicate network. Then

$$R_1(RHSL_n, x) = (4n + 2)x^{12} + (6n^2 + 4n - 4)x^9 + (6n^2 - 8n + 2)x^6. \quad (1)$$
Proof: (1) Using the partition of the revan edge set, we can apply the formula of the first Revan polynomial of $G$. Since

$$R_1(G, x) = \sum_{u \in E(G)} x^{c_u + c_1},$$

this implies that

$$R_1(RHSL_n, x) = (4n + 2)x^{12} + \left(6n^2 + 4n - 4\right)x^9 + \left(6n^2 - 8n + 2\right)x^6.$$  \hfill (2)

(2) Using the partition of the revan edge set, we can apply the formula of the second Revan polynomial of $G$.

Since

$$R_2(G, x) = \sum_{u \in E(G)} x^{c_u + c_1},$$

this implies that

$$R_2(RHSL_n, x) = (4n + 2)x^{36} + \left(6n^2 + 4n - 4\right)x^{18} + \left(6n^2 - 8n + 2\right)x^6.$$ \hfill (3)

(3) Using the partition of the revan edge set, we can apply the formula of the third Revan index of $G$

Since

$$R_3(G, x) = \sum_{u \in E(G)} x^{c_u + c_1},$$

this implies that

$$R_3(RHSL_n, x) = (4n + 2)x^{0} + \left(6n^2 + 4n - 4\right)x^{3} + \left(6n^2 - 8n + 2\right)x^{0}.$$  

$$= \left(6n^2 + 4n - 4\right)x^{3} + \left(6n^2 - 4n + 4\right).$$

Results for Rhombus Oxide Networks

We consider a family of rhombus oxide networks. A rhombus oxide network of dimension $n$ is denoted by $RHOX_n$. A rhombus oxide network of dimension 3 is depicted in Figure 2.

![Figure 2. Rhombus oxide network of dimension 3.](image)

In the following theorem, we compute the first Revan vertex index and its polynomial of rhombus oxide network.

**Theorem 4.** Let $RHOX_n$ be the rhombus oxide network. Then

$$R_0(RHOX_n) = 12n^2 + 56n.$$ \hfill (1)
\[ R_0(RHOX_n, x) = 4nx^{16} + (3n^2 - 2n)x^4. \]  

(2)

**Proof:** Let \( H \) be the graph of rhombus oxide network. By calculation, we obtain that \( H \) has \( 3n^2 + 2n \) vertices. From Figure 2, it is easy to see that the vertices of \( RHOX_n \) are either of degree 2 or 4. Thus \( \Delta(H)=4 \) and \( \delta(H)=2 \). We partition vertex set \( V(RHOX_n) \) into two sets, vertices of degree 2 and 4 respectively.

\[ V_2 = \{ u \in V(H) \mid d_H(u) = 2 \}, \quad |V_2| = 4n. \]

\[ V_4 = \{ u \in V(H) \mid d_H(u) = 4 \}, \quad |V_4| = 3n^2 - 2n. \]

Clearly, we have \( \Delta(H) + \delta(H) = 6 \). Thus \( r_{2d}(u) = 6 - d_H(u) \). Thus there are two types of Revan vertices as follows:

\[ V_{24} = \{ u \in V(H) \mid r_{2d}(u) = 4 \}, \quad |V_{24}| = 4n. \]

\[ V_{12} = \{ u \in V(H) \mid r_{2d}(u) = 2 \}, \quad |V_{12}| = 3n^2 - 2n. \]

(1) Compute \( R_0(RHOX_n) \), we see that

\[ R_0(H) = \sum_{u \in V(H)} r_H(u)^2 = \sum_{u \in V_1} r_H(u)^2 + \sum_{u \in V_4} r_H(u)^2 \]

\[ = 4n \times 4^2 + (3n^2 - 2n) \times 2^2 = 12n^2 + 56n. \]

(2) To compute \( R_0(RHOX_n, x) \), we see that

\[ R_0(H, x) = \sum_{u \in V(H)} x^{c_H(u)^2} = \sum_{u \in V_1} x^{c_H(u)^2} + \sum_{u \in V_4} x^{c_H(u)^2} \]

\[ = 4nx^4 + (3n^2 - 2n) x^2 = 4nx^{16} + (3n^2 - 2n)x^4. \]

In the following theorem, we compute the value of \( R_1(RHOX_n), R_2(RHOX_n), R_3(RHOX_n) \) for rhombus oxide networks.

**Theorem 5.** Let \( RHOX_n \) be the rhombus oxide network. Then,

\[ R_1(RHOX_n) = 24n^2 + 16n. \]  

(1)

\[ R_2(RHOX_n) = 24n^2 + 32n + 8. \]  

(2)

\[ R_3(RHOX_n) = 16n - 8. \]  

(3)

**Proof:** Let \( H \) be the rhombus oxide network. By calculation, we obtain that \( H \) has \( 6n^2 \) edges. In \( RHOX_n \), by algebraic method, there are three types of edges based on the degree of the end vertices of each edge follows:

\[ E_{22} = \{ uv \in E(H) \mid d_H(u) = d_H(v) = 2 \}, \quad |E_{22}| = 2. \]

\[ E_{24} = \{ uv \in E(H) \mid d_H(u) = 2, \ d_H(v) = 4 \}, \quad |E_{24}| = 8n - 4. \]

\[ E_{44} = \{ uv \in E(H) \mid d_H(u) = d_H(v) = 4 \}, \quad |E_{44}| = 6n^2 - 8n + 2. \]

Clearly we have \( \Delta(H)=4 \) and \( \delta(H)=2 \). Thus \( r_{2d}(u) = 6 - d_H(u) \). Thus there are three types of Revan edges based on the degree of the end revan vertices of each revan edge as follows:

\[ RE_{44} = \{ uv \in E(H) \mid r_{2d}(u) = r_{2d}(v) = 4 \}, \quad |RE_{44}| = 2. \]

\[ RE_{42} = \{ uv \in E(H) \mid r_{2d}(u) = 4, \ d_H(v) = 2 \}, \quad |RE_{42}| = 8n - 4. \]

\[ RE_{22} = \{ uv \in E(H) \mid r_{2d}(u) = r_{2d}(v) = 2 \}, \quad |RE_{22}| = 6n^2 - 8n + 2. \]

(1) To compute \( R_1(RHOX_n) \), we see that
\( R_i(H) = \sum_{u \in E(H)} [r_I(u) + r_I(v)] = \sum_{E_{uv}} [r_I(u) + r_I(v)] + \sum_{E_{uv}} [r_I(u) + r_I(v)] + \sum_{E_{uv}} [r_I(u) + r_I(v)] \)

\[ = 8 \times 2 + 6(8n - 4) + 4(6n^2 - 8n + 2) = 24n^2 + 16n. \]

(2) To compute \( R_2(RHOX_n) \), we see that

\[ R_2(H) = \sum_{u \in E(H)} r_I(u)r_I(v) = \sum_{E_{uv}} r_I(u)r_I(v) + \sum_{E_{uv}} r_I(u)r_I(v) + \sum_{E_{uv}} r_I(u)r_I(v) \]

\[ = 16 \times 2 + 8(8n - 4) + 4(6n^2 - 8n + 2) = 24n^2 + 32n + 8. \]

(3) To compute \( R_3(RHOX_n) \), we see that

\[ R_3(H) = \sum |r_I(u) - r_I(v)| = \sum |r_I(u) - r_I(v)| + \sum |r_I(u) - r_I(v)| + \sum |r_I(u) - r_I(v)| \]

\[ = 0 \times 2 + 2(8n - 4) + 0(6n^2 - 8n + 2) = 16n - 8. \]

In the following theorem, we compute the value of \( R_i(RHOX_n, x), R_2(RHOX_n, x), R_3(RHOX_n, x) \) for rhombus oxide networks.

**Theorem 6.** Let \( RHOX_n \) be the rhombus oxide network. Then

\[ R_1(RHOX_n, x) = 2x^8 + (8n - 4)x^6 + (6n^2 - 8n + 2)x^4. \]  
(1)

\[ R_2(RHOX_n, x) = 2x^{16} + (8n - 4)x^8 + (6n^2 - 8n + 2)x^4. \]  
(2)

\[ R_3(RHOX_n, x) = (8n - 4)x^2 + 6n^2 - 8n + 4. \]  
(3)

**Proof:** (1) Using the partition of the revan edge set, we can apply the formula of the first Revan polynomial of \( H \).

Since \( R_1(H, x) = \sum_{u \in E(H)} x^{v(u)} r_e(v) \),

this implies that

\[ R_1(RHOX_n, x) = 2x^8 + (8n - 4)x^6 + (6n^2 - 8n + 2)x^4. \]

(2) Using the partition of the revan edge set, we can apply the formula of the second Revan polynomial of \( H \).

Since \( R_2(H, x) = \sum_{u \in E(H)} x^{v(u)} r_e(v) \),

this implies that

\[ R_2(RHOX_n, x) = 2x^{16} + (8n - 4)x^8 + (6n^2 - 8n + 2)x^4. \]

(3) Using the partition of revan edge set, we can apply the formula of the third Revan polynomial of \( H \).

Since \( R_3(H, x) = \sum_{u \in E(H)} x^{v(u)} r_e(v) \),

this implies that

\[ R_3(RHOX_n, x) = 2x^2 + (8n - 4)x^2 + (6n^2 - 8n + 2)x^0. \]

\[ = (8n - 4)x^2 + 6n^2 - 8n + 4. \]

**REFERENCES**


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